Table A1

Annular channel With heat input at inner wall	Annular channel With heat input at outer wall	Hydraulically equivalent circular channel
$q''_{ACi} = \dfrac{q'_w}{2\pi R_i}$	$q''_{ACo} = \dfrac{q'_w}{2\pi R_o}$	$q''_{ECC} = \dfrac{q'_w}{2\pi(R_o - R_i)}$

where

$$X = 1 - \exp(-0.48 \times 10^5/Pe)$$

and

$$Y = 1 - \exp(-0.98 \times 10^{-5}/Pe).$$

That this criterion is valid for tubular channels of various sizes is borne out to a certain extent by the size range considered in the development of the criterion. Let us now replace the annular channel by a hydraulically equivalent tubular channel of diameter $2(R_o - R_i)$. Keeping the heat input rate per unit channel length, q'_w, the same, Table A1 gives the situations *vis-à-vis* wall heat flux. Therefore

$$\frac{q''_{ACi}}{q''_{ECC}} = \frac{R_o - R_i}{R_i} = \gamma_i$$

and

$$\frac{q''_{ACo}}{q''_{ECC}} = \frac{R_o - R_i}{R_o} = \gamma_o.$$

We propose that the wall heat flux, q''_w, which appears in equation (A1) be replaced by q''_{wECC}. Note that if the test section is tubular, q''_{wECC} is the actual wall heat flux imposed and equation (A1) remains the same. It, however, is modified in the case of annular geometry

$$(\Delta T_{sub})_{NVG} = \frac{q''_{wECC}}{[(455k_L/D_h)X + (0.0065GC_p)Y]} = \frac{q''_{wACj}}{\gamma_j[(455k_L/D_h)X + (0.0065GC_p)Y]} \quad (A2)$$

where j = i or o.

Note that $\gamma_j > 1$ causes the predicted NVG location to move downstream in the channel (with respect to the location where heating begins) provided NVG does not occur at the beginning of the heating in both cases. On the other hand, $\gamma_j < 1$ causes the predicted NVG location to move upstream.

The preceding modification suggested for high Peclet number ($\approx 10^5$ or higher) flows is tentative since its verification is rather limited. However, it appears to be reasonable to propose that γ_j (j = i or o) be adopted as the upper or lower (as appropriate) limit of a correction parameter β_j such that

$$1 \leqslant \beta_i \leqslant \gamma_i \quad (\text{if } \gamma_i > 1)$$

$$\gamma_i \leqslant \beta_i \leqslant 1 \quad (\text{if } \gamma_i < 1)$$

and

$$1 \geqslant \beta_o \geqslant \gamma_o.$$

The optimum value of β_j (within the above ranges) for a particular annular channel should be decided upon by flow visualization when feasible. When this is not possible, indirect optimization via comparison of modal-calculated axial vapor fraction profiles with corresponding experimental data could be used. For our test section, we estimated the following value of β_i to be suitable on the basis of flow visualization:

$$\beta_i = 1.25 \pm 0.05.$$

In the calculational result of Tables 1 and 2, $\beta_i = 1.25$ has been used.

Int. J. Heat Mass Transfer. Vol. 31, No. 9, pp. 1952–1954, 1988
Printed in Great Britain
0017-9310/88 $3.00+0.00
Pergamon Press plc

The use of the Walsh functions for the study of periodic diffusive phenomena in a multilayer medium

G. GUIFFANT, A. ARHALIASS and J. DUFAUX
L.B.H.P., Université Paris 7, 2 place Jussieu, 75251 Paris Cedex 05, France

(*Received* 3 *December* 1987)

INTRODUCTION

A VERY simple method for studying transient diffusive phenomena in multilayer media was proposed by Gosse [1]. This approach is extended to periodic external conditions by using developments on an orthogonal complete base of rectangular functions called 'Walsh functions'. We take advantage of this note to emphasize the interest of the problem both for practical applications and for the methods used. The study of the transport phenomena in the microcirculation is then pointed out as a particular example of the possible transfer of methods by means of analogies.

The main features of the transient diffusive phenomena in heterogeneous media are both dependent on the nature of the constitutive materials and on their geometrical repartition. Thus, two classes of media can be taken into consideration: (a) composite materials which are constituted of a solid dispersed phase of regular or irregular shape elements; (b) media made up of successive ajoining layers of different materials.

Owing to the possible applications, the study of the diffusion in multilayer media occurs in situations where the system is submitted (at least on one external face) to periodic limiting conditions: as an example, in the field of thermal study, we use here as a point of reference, the alternate watch and fire temperature or the day–night succession (thermal analysis of stratified soils, thermal energy storage, isolating walls, etc.).

According to the preceding statement, the problem gives the opportunity to develop methods of resolution allowing, by means of analogies, to start on any other question defined in a similar way. From this point of view, the development of models for the study of transport phenomena in the microcirculation is mentioned here [2]. In the microvascular bed, blood flow is mainly controlled by viscous forces. Thus, it can be shown [3] that the intravascular pressure is a solution of a diffusion equation with a diffusivity as a function of the blood viscosity and characteristic parameters of the vessel: dimensions and Young's modulus. The observation *in situ* of microcirculatory networks (double tree vascularization of vessels with decreasing and increasing diameters) leads to postulate [3] a representation consisting of a number of levels connected in series, each level being characterized by constant and uniform parameters (same geometric parameters, viscosity and Young's modulus). Moreover, the equation for the blood flow continuity between two levels [3] is analogous

NOMENCLATURE

A, B	coefficients of development in equation (3)	η	eigenvalue
a	diffusivity	θ	temperature
C	heat capacity	λ	thermal conductivity
e	thickness	ν, δ	Walsh coefficients in equation (13)
T	temperature	τ	period
x	longitudinal coordinate.	ϕ	heat flux.

Greek symbols

α	Walsh coefficient in equation (9)
β	dimensionless coefficient in equation (4)
Γ	product of matrices defined by equation (6)
$\varepsilon, \eta, \xi, \chi$	terms of matrix (6)

Subscripts

i,j	related to one slab
k	order of an elementary solution
n	total number of slabs
p	order of Walsh function.

to the equation of continuity for the heat flux between two slabs for the thermal problem. At least, it is clear that a periodic limiting condition is to be applied at the entrance of such a multilayer system as a result of the persistence of the cardiac pulse.

The problem of the transient heat conduction in a multilayer medium has been solved by Gosse [1] with a simple and original method. The aim of the present note is to add a complement to this analytical approach which enables one to solve the problem for any periodic limiting conditions. We will briefly recall the essential points of the method suggested by Gosse, then we will show how the solutions obtained, for imposed steps of temperature, on an external face, can be transformed to obtain the solution for any periodic limiting condition of temperature.

DESCRIPTION OF THE METHOD

We consider a multilayer medium made up of n adjacent slabs with constant and uniform parameters: density ρ_i, thermal conductivity λ_i, heat capacity C_i, diffusivity a_i, thickness e_i. For a slice i, the temperature T_i is a solution of the diffusion equation

$$\frac{\partial T_i}{\partial t} = a_i \frac{\partial^2 T_i}{\partial x_i^2}. \tag{1}$$

We suppose both the temperature and the heat flux to be continuous at each interface (no contact resistances).

The problem can be greatly simplified by using a particular dimensional transformation. Taking the first slab ($i = 1$) as a reference

$$x_i = e_i \sqrt{\left(\frac{a_i}{a_1}\right)} x_i^*; \quad t = \frac{e_1^2}{a_1} t^* \tag{2}$$

the asterisk * will be omitted for simplification.

By using the separation of variables method, one can obtain an elementary solution of order k in the form

$$T_{ik} = [A_{ik} \cos(\mu_k x_i) + B_{ik} \sin(\mu_k x_i)] \, \mathrm{e}^{-\mu_k^2 t} \tag{3}$$

$$\phi_{ik} = \beta_i \mu_k [A_{ik} \sin(\mu_k x_i) - B_{ik} \cos(\mu_k x_i)] \, \mathrm{e}^{-\mu_k^2 t} \tag{4}$$

where ϕ_i is the heat flux and

$$\beta_i = \frac{\lambda_i}{e_i} \sqrt{\left(\frac{a_1}{a_i}\right)} \left(\sum_i \frac{e_i}{\lambda_i}\right).$$

Equation (2) gives a unique reduced time for all the slabs of the wall. Then, by using the conditions of continuity at each interface, the solution of order k in the slab i can be obtained in the form

$$\begin{bmatrix} T_{ik}(e_i, t) \\ \phi_{ik}(e_i, t) \end{bmatrix} = \Gamma_{ik} \begin{bmatrix} T_{ik}(0, t) \\ \phi_{ik}(0, t) \end{bmatrix} \tag{5}$$

$$\Gamma_{ik} = \prod_i^1 \begin{bmatrix} \cos(\mu_k e_j) & -\dfrac{\sin(\mu_k e_j)}{\mu_k \beta_j} \\ \mu_k \beta_j \sin(\mu_k e_j) & \cos(\mu_k e_j) \end{bmatrix} = \begin{bmatrix} \xi_{ik} & \eta_{ik} \\ \zeta_{ik} & \chi_{ik} \end{bmatrix}. \tag{6}$$

When imposing constant temperatures on the external faces, the set of eigenvalues μ_k is the solution of the transcendental equation

$$\eta_{nk} = 0. \tag{7}$$

By using the aforementioned procedure it becomes easy to study the thermal behaviour of a multilayer wall submitted on one face to a series of periodic sequences of temperature $T_n(e_n, t)$ [4]. The solution in slab i is

$$T_i(x_i, T) = \sum_k T_{ik}(x, t). \tag{8}$$

With a suitable choice of the series of sequences, it is then possible to identify $T_n(e_n, t)$ with one term of the sequential analysis of a signal of temperature (with period τ) on a complete orthogonal set of rectangular functions. The Walsh functions [5] are a convenient set for such a development

$$T_n(e_n, t) \equiv \alpha_p \, \mathrm{Wal}\,(p, t) \tag{9}$$

where α_p is the coefficient of the development of order p. p is the order of the Walsh function Wal (p, t).

Solution (8) can be written in the form

$$T_i^p(x_i, t) = \sum_k T_{ik}^p(x_i, t). \tag{10}$$

Let $\theta_n(t)$ be the periodic signal of temperature applied on one external face of the system

$$\theta_n(t) = \sum_p \alpha_p \, \mathrm{Wal}\,(p, t) \tag{11}$$

in accordance with the principle of superposition, the general solution in slab i can be written in the form

$$\theta_i(x_i, t) = \sum_p T_i^p(x_i, t). \tag{12}$$

By analogy with the Fourier developments, the Walsh functions are divided into two groups noted 'Cal' and 'Sal' according to their parity with respect to the half period ($\tau/2$). Then

$$\theta_n(t) = \nu_0 \, \mathrm{Wal}\,(0, t) + \sum_p [\nu_p \, \mathrm{Cal}\,(p, t) + \delta_p \, \mathrm{Sal}\,(p, t)] \tag{13}$$

$$\nu_0 = \frac{1}{\tau} \int_0^\tau \theta_n(t) \, \mathrm{d}t; \quad \nu_p = \frac{1}{\tau} \int_0^\tau \theta_n(t) \, \mathrm{Cal}\,(p, t) \, \mathrm{d}t;$$

$$\delta_p = \frac{1}{\tau} \int_0^\tau \theta_n(t) \, \mathrm{Sal}\,(p, t) \, \mathrm{d}t. \tag{14}$$

As an example, we have shown in Fig. 1 the first four Walsh functions.

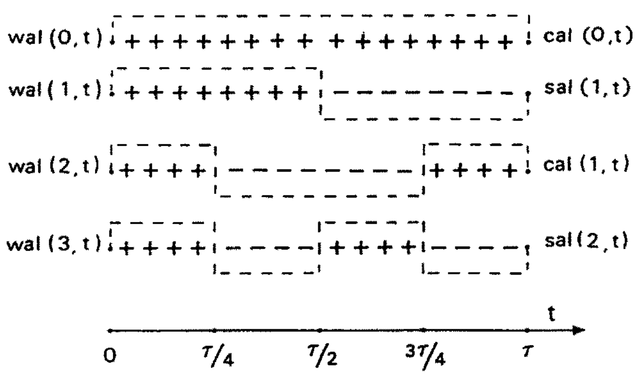

FIG. 1. The first four Walsh functions.

The sequential development of the imposed condition on the outer face is clearly liable to a simple physical interpretation : a series of sequences of temperature of given duration. Thus the sequential analysis appears to be a suitable tool for the study of the diffusive phenomena (the frequency Fourier analysis remaining the essential method for the study of the oscillatory phenomena).

The suggested approach provides a powerful method for the study and the simulation of the diffusive transport phenomena in multilayer media [6] particularly when numerous slabs are involved with parameters of different orders of magnitude. The whole procedure leads to analytical solutions which appear here to be more precise and easier to obtain, with a computer, than when using a finite difference procedure because of the restriction on the time step as a function of the mesh size. In fact, the method can only be limited by the computer itself. The greatest limitation comes from the rounding error, which can become important when a great number of terms is necessary for the reconstruction of the solution at a given point of the medium. In such a case (rapidly varying limiting conditions) the computing time could become important and additional memories or special programming procedures would be necessary.

REFERENCES

1. J. Gosse, Sur la conduction thermique variable dans un mur composite à propriétés constantes, *C.r. Hebd. Séanc. Acad. Sci., Paris* **B286**, 303–306 (1978).
2. A. Arhaliass, G. Guiffant et J. Dufaux, L'utilisation des fonctions de Walsh pour l'étude des phénomènes diffusifs, en régime périodique, dans des milieux multicouches. Application à la microcirculation, Forum des Jeunes Physiciens, Oléron (1987) (SFP).
3. J. Gross, M. Intaglietta and B. W. Zweifach, Network model of pulsatile hemodynamics in the microcirculation of the rabbit omentum, *Am. J. Physiol.* **226,** 5 (1974).
4. P. Duhamel et J. Gosse, Analyse thermique d'un mur composite soumis sur une face à une variation en créneaux de la température, *Int. J. Heat Mass Transfer* **23,** 1663–1671 (1980).
5. J. Liferman, *Les méthodes rapides de transformation du signal.* Masson, Paris (1980).
6. A. Arhaliass, J. Dufaux, G. Guiffant et J. J. Durussel, Simulation de l'évolution du champ de pression (ou de vitesse) dans un réseau de microvaisseaux modélisé par une structure en couche, soumis à I.T.B.M., Paris (mai 1988).